Math 141 Sample Problems for Midterm Exam

Question 1 Determine the following limits (if they exist):

a) $\lim_{x\to -2} \dfrac{x-|x^3-4|}{|x|+3}$

b) $\lim_{x\to 4} \dfrac{x^2-16}{\sqrt{x}-2}$

c) $\lim_{x\to 4^-} \dfrac{\sqrt{x+5}-4}{x^2-2x-8}$

d) $\lim_{x\to 0} \dfrac{\sin 2x}{4x}$

e) $\lim_{x\to 0} \dfrac{1-\cos 3x}{2x^2}$

f) $\lim_{x\to \pi} \dfrac{\sin x}{x-\pi}$

Question 2 Find a value of A so that the function $f(x)$ is continuous for all values of x.

$$f(x) = \begin{cases} \cos(\pi x) + \sqrt{4-x} & \text{if } x \le 2 \\ \sqrt{2} + \sin(A\pi x) & \text{if } x > 2 \end{cases}$$

Question 3 Show that the equation $\sqrt[3]{x+1} = 3 - x^2$ has at least one solution on the interval $[0, \infty)$.

Question 4

a) Use the (limit) definition of derivative to compute the derivative $f'(x)$ for the function $f(x) = \dfrac{3x-1}{2x+3}$.

b) Find the equation of the tangent line to $y = \dfrac{3x-1}{2x+3}$ at the point $x = 3$.

Question 5 Compute the derivatives of the following functions (do not simplify):

a) $f(x) = 4x^3 - \sqrt[3]{x}$

b) $f(x) = \left(3x^2 - \dfrac{4}{x^2}\right)(3x^7 - 8x^4 + 2x^{-1/4})$

c) $f(x) = \dfrac{x^2 - \sqrt{x}}{5x + 3x^{-2}}$

d) $f(x) = (3x^3 - 4x + 9)^{7/3}$

e) $f(x) = \cos^7\left(4x^2 - \dfrac{5}{\sqrt{x}}\right)$

Question 6 Find the value(s) of x where the graph of $y = 3x^{2/3} + 4x$ has a horizontal tangent line.

Question 7 For the function $g(t) = 4 - 3t + t^3$. Find the average rate of change of $g(t)$ over the interval $[-2, 3]$. What is the instantaneous rate of change of $g(t)$ at $t = 1$?

Question 8 Find the equation of the tangent line to the curve defined by $x^2y + x\sin(x-y) = \pi/2$ at the point $(\frac{\pi}{2}, 0)$.